A **financial instrument** is a legal contract between two or more parties that defines conditions under which the various parties incur costs and receive benefits. A cost or benefit need not be a monetary amount; it could be a commodity, for instance. The simplest type of financial instrument is a **financial asset**, which is a legal claim on a real asset such as a company, a commodity, cash, gold or a building. A **financial security** is a standardized form of financial asset that is traded in an organized market. For instance, equity securities (shares on a company’s stock) are traded on exchanges and debt securities such as bonds and money market instruments (including bills, notes and repurchase agreements) are traded in brokers’ markets.

A **derivative contract**, usually called a ‘derivative’ for short, is another type of financial instrument which is a contract on one or more **underlying** financial instruments. The underlying of a derivative does not have to be a traded asset or an interest rate. For instance, futures on carbon emissions or temperature have started trading on exchanges during the last few years. Derivatives are the fastest-growing class of financial instruments and the notional amount outstanding now far exceeds the size of ordinary securities markets. For instance, in 2007 the Bank for International Settlements estimated the total size of the debt securities market (including all corporate, government and municipal bonds and money market instruments) to be approximately US$70 trillion. However, the amount outstanding on all interest rate derivatives was nearly $300 trillion.

The most common types of financial derivatives are futures and forwards, swaps and options, and within each broad category there are numerous subcategories, so there is a huge diversity of financial derivatives. For instance, the vast majority of the trading in swaps is on interest rate swaps, but credit default swaps and cross-currency basis swaps are also heavily traded. Other swaps include variance swaps, covariance swaps, equity swaps and contracts for differences. But the greatest diversity amongst all derivative instruments can be found in the category of options. Options can be defined on virtually any underlying contract, including options on derivatives such as futures, swaps and other options. Many options, mostly standard calls and puts, are traded on exchanges, but there is a very active over-the-counter (OTC) market in non-standard options. Since the two parties in an OTC contract are free to define whatever terms they please, the pay-off to the holder of an OTC option can be freely defined. This means that ever more exotic options are continually being introduced, with pay-off profiles that can take virtually any shape.

A **portfolio** is a collection of financial instruments. An investor holds a portfolio with the aim of obtaining a particular return on his investment and to spread his risk. The more
differences between the financial instruments available to the investor, the better he can diversify his risk. Risk can be substantially reduced in large, well-diversified portfolios, but there can never be zero risk associated with any return above the risk free rate, and some investors are more averse to taking risks than others. The main reason for the terrific number of different financial instruments is that the risk–return profiles of different investors are not the same. Each new type of instrument is introduced specifically because it purports to provide its own unique profile of risk and return.

AIMS AND SCOPE

This book is designed as a text for advanced university and professional courses in finance. It provides a pedagogical and complete treatment of the characteristics of the main categories of financial instruments, i.e. bonds, swaps, futures and forwards, options and volatility. Given the tremendous diversity of financial instruments, it is not surprising that there are many books that deal with just one type of financial instrument. Often the textbooks that cover fixed income securities alone, or just futures and forwards, or swaps or options, are large books that go into considerable details about specific market conventions. Some present each subcategory of instrument in its own unique theoretical framework, or include all mathematical details. By contrast, this book adopts a general framework whenever possible and provides a concise but rigorous treatment of only the essential mathematics.

To cover all major financial instruments (excluding credit derivatives) in one volume, one has to be very selective in the material presented. The reason why I have decided to exclude credit derivatives is that this book series is on market risk and not credit risk. Also I have not set up the background in Volume I, Quantitative Methods in Finance, to be able to cover credit derivatives in the same detail as I can analyse swaps, futures, options and volatility. Also we do not have a chapter specifically devoted to cash equity in this volume. This material naturally belongs in the Econometrics volume of Market Risk Analysis. A large part of Volume II, Practical Financial Econometrics, concerns cash equity portfolios, including the regression factor models that are used to analyse their risk and return and more advanced equity trading strategies (e.g. pairs trading based on cointegration).

Readers will appreciate the need to be concise, and whilst a mathematically rigorous approach is adopted some detailed proofs are omitted. Instead we refer readers to tractable sources where proofs may be perused, if required. My purpose is really to focus on the important concepts and to illustrate their application with practical examples. Even though this book omits some of the detailed arguments that are found in other textbooks on financial instruments, I have made considerable effort not to be obscure in any way. Each term is carefully defined, or a cross-reference is provided where readers may seek further enlightenment in other volumes of Market Risk Analysis. We assume no prior knowledge of finance, but readers should be comfortable with the scope of the mathematical material in Volume I and will preferably have that volume to hand. In order to make the exposition accessible to a wide audience, illustrative examples are provided immediately after the introduction of each new concept and virtually all of these examples are also worked through in interactive Excel spreadsheets.

This book is much shorter than other general books on financial instruments such as Wilmott (2006), Hull (2008) and Fabozzi (2002), one reason being that we omit credit derivatives. Many other textbooks in this area focus on just one particular category
of financial instrument. Thus there is overlap with several existing books. For instance, Chapter 3 on Options covers the same topics as much of the material in James (2003). A similar remark applies to Gatheral (2006), which has content similar to the first 75 pages of Chapter 4, on Volatility but in Gatheral’s book this is covered in greater mathematical depth.

The readership of this volume is likely to be equally divided between finance professionals and academics. The main professional audience will be amongst traders, quants and risk managers, particularly those whose work concerns the pricing and hedging of bonds, swaps, futures and forwards, options and volatility. The main academic audience is for faculty involved with teaching and research and for students at the advanced master’s or PhD level in finance, mathematical finance or quantitative market risk management. There are only five (extremely long) chapters and each aims to provide sufficient material for a one-semester postgraduate course, or for a week’s professional training course.

OUTLINE OF VOLUME III

Chapter 1, Bonds and Swaps, begins by introducing fundamental concepts such as the compounding of interest and the relationship between spot and forward rates, by providing a catalogue of fixed and floating coupon bonds by issuer and maturity and by performing a basic analysis of fixed coupon bonds, including the price–yield relationship, the characteristics of the zero coupon spot yield curve and the term structure of forward interest rates. We cover duration and convexity for single bonds and then for bond portfolios, the Taylor expansion to approximate the change in portfolio price for a parallel shift in the yield curve, and the traditional approach to bond portfolio immunization. Then we look at floating rate notes, forward rate agreements and interest rate swaps and explain their relationship; we analyse the market risk of an interest rate swap and introduce the PV01 and the dollar duration of cash flow. Bootstrapping, splines and parametric yield curve fitting methods and convertible bonds are also covered in this chapter.

Chapter 2, Futures and Forwards, gives details of the futures and forward markets in interest rates, bonds, currencies, commodities, stocks, stock indices, exchange traded funds, volatility indices, credit spreads, weather, real estate and pollution. Then we introduce the no arbitrage pricing argument, examine the components of basis risk for different types of underlying contract, and explain how to hedge with futures and forwards. Mean–variance, minimum variance and proxy hedging are all covered. We illustrate how futures hedges are implemented in practice: to hedge international portfolios with forex forwards, stock portfolios with index futures, and bond portfolios with portfolios of notional bond futures. The residual risk of a hedged portfolio is disaggregated into different components, showing which uncertainties cannot be fully hedged, and we include an Excel case study that analyses the book of an energy futures trader, identifying the key risk factors facing the trader and providing simple ways for the trader to reduce his risks.

Chapter 3, Options, introduces the basic principles of option pricing, and the options trading strategies that are commonly used by investors and speculators; describes the characteristics of different types of options; explains how providers of options hedge their risks; derives and interprets the Black–Scholes–Merton pricing model, and a standard trader’s adjustment to this model for stochastic volatility; explains how to price interest rate options and how to calibrate the LIBOR model; and provides pricing models for European exotic options. It
begins with a relatively non-technical overview of the foundations of option pricing theory, including some elementary stochastic calculus, deriving the principle of risk neutral valuation and explaining the binomial option pricing model. The scope of the chapter is very broad, covering the pricing of European and American options with and without path-dependent pay-offs, but only under the assumption of constant volatility. ‘Greeks’ are introduced and analysed thoroughly and numerical examples how to hedge the risks of trading options. For interest rate options we derive the prices of European caps, floors and swaptions and survey the family of mean-reverting interest rate models, including a case study on the LIBOR model. Formulae for numerous exotics are given and these, along with more than 20 other numerical examples for this chapter, are all implemented in Excel.

Chapter 4, Volatility, begins by explaining how to model the market implied and market local volatility surfaces and discusses the properties of model implied and model local volatility surfaces. A long case study, spread over three Excel workbooks, develops a dynamic model of the market implied volatility surface based on principal component analysis and uses this to estimate price hedge ratios that are adjusted for implied volatility dynamics. Another main focus of the chapter is on option pricing models with stochastic volatility and jumps. The model implied and local volatility surfaces corresponding to any stochastic volatility model are defined intuitively and several stochastic volatility models, including their applications to options pricing and hedging, are discussed. We cover a few specific models with jumps, such as the Heston jump model (but not Lévy processes) and introduce a new type of volatility jump model as the continuous version of Markov switching GARCH. We explain why the models for tradable assets (but not necessarily interest rates) must be scale invariant and why it does not matter which scale invariant model we use for dynamic delta–gamma hedging of virtually any claim (!). Then we describe the market and the characteristics of variance swaps, volatility futures and volatility options and explain how to construct a term structure of volatility indices, using for illustration the Vfše, a volatility index that is not currently quoted on any exchange. At 94 pages, it is one of the longest and most comprehensive chapters in the book.

Chapter 5, Portfolio Mapping, is essential for hedging market risks and also lays the foundations for Volume IV, Value-at-Risk Models. It begins by summarizing a portfolio’s risk factors and its sensitivities to these factors for various categories of financial instruments, including cash and futures or forward portfolios on equities, bonds, currencies and commodities and portfolios of options. Then it covers present value, duration, volatility and PV01 invariant cash flow mapping, illustrating these with simple interactive Excel spreadsheets. Risk factor mapping of futures and forward portfolios, and that of commodity futures portfolios in particular, and mappings for options portfolios are covered, with all technical details supported with Excel spreadsheets. Mapping a volatility surface is not easy and most vega bucketing techniques are too crude, so this is illustrated with a case study based on the Vfše index. Statistical techniques such as regression and principal component analysis are used to reduce the dimension of the risk factor space and the chapter also requires some knowledge of matrix algebra for multivariate delta–gamma mapping.

ABOUT THE CD-ROM

Virtually all the concepts in this book are illustrated using numerical and empirical examples which are stored in Excel workbooks for each chapter. These may be found on the accompanying CD-ROM in the folder labelled by the chapter number. Within these spreadsheets
readers may change parameters of the problem (the parameters are indicated in red) and see the new solution (the output is indicated in blue).

Rather than using VBA code, which will be obscure to many readers, I have encoded the formulae directly into the spreadsheet. Thus the reader need only click on a cell to read the formula and it should be straightforward to tailor or extend most of the spreadsheets to suit the reader’s particular problems. Notably, they contain formulae for exotic option prices, not only barrier options and Asians but also pricing formulae for many other exotics. Matlab code, written by my PhD student Andreas Kaeck, is provided for calibrating option pricing models. Several case studies, based on complete and up-to-date financial data, and all graphs and tables in the text are also contained in the Excel workbooks on the CD-ROM.

For teaching purposes, the Excel spreadsheets are designed so that the course tutor can set an unlimited number of variations on the examples in the text as exercises. Also the graphs and tables can be modified if required, and copied and pasted as enhanced metafiles into lecture notes (respecting the copyright notice that is included at the end of the book).

ACKNOWLEDGEMENTS

One of the problems with being an author is that to be truly original one should minimize contact with related textbooks. But if one possesses books only to consult them briefly, for instance to verify a formula, how does one learn? Reading academic research papers is very important of course, but most of the practical knowledge of finance and risk management that I bring to this textbook has been gained through discussions with my husband, Jacques Pézier. We share a passion for mathematical finance. The first two presents he gave me were paperweights with shapes resembling a volatility surface and a normal mixture copula density. When I met Jacques I was a mere econometrician having some expertise with GARCH models, but because of him I have moved into mainstream quantitative finance, a change that has been continually fuelled by our frequent discussions. I can honestly say that without Jacques my state of knowledge would not warrant writing this book and it gives me enormous pleasure to dedicate it to him.

Jacques spent twenty-five years working in the City as a consultant and a financial risk manager, helping to set up LIFFE and to build risk management groups for several major banks. And his hand-written documents for the original version of Reuters 2000 software in 1994 formed the basis of the exotic option spreadsheets included on the CD-ROM. Five years ago I eventually persuaded him to return to academic life, and now we work side by side at the ICMA Centre with a large and wonderful quantitative finance research group. I would like to thank Professor John Board, Director of the Centre, and the two past directors, Professors Brian Scott-Quinn and Chris Brooks, for creating an environment in which this is possible.

I would like to thank my very careful copyeditor, Richard Leigh, who has been good-humoured and patient with my last minute changes to the text. It helps so much to have a specialist mathematical copyeditor who can spot errors in equations, and Richard is also an excellent linguist. He is very much appreciated, not only by me but also by Viv Wickham, whom I would like to thank for the lightening speed and efficiency with which she published these books, and all her staff on the production side at Wiley.

Like most academics, I choose research problems because I want to learn more about a certain area, and it is so much more pleasurable to walk the path of learning accompanied
by a student. My PhD students have played a very important role in the advancement of my knowledge and I have been lucky enough to supervise a succession of excellent students at the ICMA Centre. All of these students, past and present, have contributed significantly to my understanding of mathematical finance and quantitative risk management. Those whose research relates to this book deserve a special mention. My research with Dr Ali Bora Yigitbaşoğlu, who now works at Lehman Brothers in London, contributed to my understanding of convertible bond pricing and hedging models. I learned how to build yield curves and calibrate the LIBOR model with Dr Dmitri Lvov, now at JP Morgan Chase in London. With Dr Andreza Barbosa, who is also working at JP Morgan Chase, I learned about exchange traded finds and minimum variance hedging with futures. And with Aanand Venkatramanan, who is now in the third year of his PhD, I learned about commodity markets and about multi-asset option pricing.

Two of my past PhD students are now highly valued colleagues at the ICMA Centre. These are Dr Emese Lazar and Dr Leonardo Nogueira. Emese and I continue to work together on the continuous limit of GARCH models, where her confidence, perseverance and meticulous calculations have led to some important breakthroughs. In particular, we have derived two new GARCH diffusions, the numerous advantages of which have been described in Chapter 4. And Leonardo’s expansive vision, energy and enthusiasm for new research problems have led to some far-reaching results on hedging options. Our papers on scale invariance have cut through a considerable research effort on finding the best hedging model, since we have shown that all appropriate models for hedging any options on tradable assets have the same hedge ratios! Working with Leonardo, I learned a considerable amount about volatility modelling.

My good friends Dr Hyungsok Ahn of Nomura, London and Moorad Choudhry of KBC financial products, have shared many insights with me. I am also privileged to count the two top academics in this field amongst my friends, and would like to extend my thanks to Professor Emanuel Derman of Columbia University and Dr Bruno Dupire of Bloomberg, for their continuous support and encouragement.

Finally, I would like to express my sincere gratitude to three of my present PhD students who have been of invaluable assistance in preparing this book. Joydeep Lahiri has prepared all the three dimensional graphs in these volumes. Stamatis Leontsinis and Andreas Kaeck have spotted several errors when reading draft chapters, and Andreas has very kindly allowed readers access to his option pricing calibration code. My students count amongst their many talents an astonishing capacity for computational and empirical work and their assistance in this respect is very gratefully acknowledged.